

Roll No.

Total Pages : 04

BT-2/M-19
32001
MATHEMATICS-II
MATH-102E
Group II (Opt. II)
(Common for All Branches)

Time : Three Hours]

[Maximum Marks : 100

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) Using elementary transformations, find the inverse

of the matrix $\begin{bmatrix} 3 & 2 & 4 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$. 10

- (b) Show that the equations :

$$3x + 4y + 5z = a$$

$$4x + 5y + 6z = b$$

$$5x + 6y + 7z = c$$

do not have a solution unless $a + c = 2b$. 10

2. (a) Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}. \quad 10$$

- (b) Given that $A = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$, show that

$$(1-A)(1+A)^{-1} \text{ is a unitary matrix.} \quad 10$$

Unit II

3. (a) Solve the following equation :

$$(3x^2 + 6xy^2)dx + (6x^2y + 4y^3)dy = 0 \quad 10$$

- (b) If the temperature of the air is 30°C and the substance cools from 100°C to 70°C in minutes, find when the temperature will be 40°C. 10

4. (a) Solve the equation :

$$\frac{d^2y}{dx^2} + 4y = x^2 + \cos 2x \quad 10$$

- (b) Solve :

$$x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + y = \frac{1}{(1-x)^2} \quad 10$$

Unit III

5. (a) Find the Laplace transform of :

(i) $\frac{1-e^t}{t}$

(ii) $|t-3|+|t+3|, t \geq 0.$ 10

(b) Find the inverse transform of $\frac{s^2+s-2}{s(s+3)(s-2)}$. 10

6. (a) State and prove the convolution theorem of Laplace transform. 10

(b) Solve the following equation by transform method, $y'' + 4y' + 3y = e^{-t}, y(0) = y'(0) = 1.$ 10

Unit IV

7. (a) Form the partial differential equation from :

$$xyz = \phi(x+y+z) \quad 10$$

(b) Solve the PDF :

$$x^2(y-z)p + y^2(z-x)q = z^2(x-y) \quad 10$$

8. (a) Using Charpit's method, solve :

$$2xy - px^2 - 2qxy + pq = 0 \quad 10$$

(b) Solve the following equations by the method of separation of variables :

$$3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0; \quad u(x, 0) = 4e^{-x}.$$